Fracciones algebraicas - Ecuaciones racionales

FRACCIÓN : $\boxed{\frac{P(x)}{Q(x)}}$

1) SIMPLIFICAR UNA FRACCIÓN : FACTORIZAMOS POLINOMIOS .

$$\frac{x^2-x-6}{x^2-2x-3}$$

$x^2-x-6=0$; $x=\frac{1\pm\sqrt{1+24}}{2}=\frac{1\pm5}{2}$

$x=3$; $x=-2$

$(x-3)(x+2)$

$x^2-2x-3=0$; $x=\frac{2\pm\sqrt{4+12}}{2}=$

$=\frac{2\pm4}{2}$; $x=3$; $x=-1$

$(x-3)(x+1)$

$$\frac{(x-3)(x+2)}{(x-3)(x+1)}=\boxed{\frac{x+2}{x+1}}$$

$$\frac{x^3-2x^2-x+2}{x^3-7x+6}=\frac{x+1}{x+3}$$

2) SUMA Y/O RESTA FRACCIONES (mcm)

(OPERACIÓN).

$$\frac{3x}{x^2-1}+\frac{2x}{x+1}-\frac{5}{x-1}=$$

$x^2-1^2=0$

$x=\pm1$; $(x+1)(x-1)$

$\frac{3x}{(x+1)(x-1)}+\frac{2x}{x+1}-\frac{5}{x-1}$ ⇝ mcm = $(x+1)\cdot(x-1)$

(Hacia Denominador)

$$\left(\frac{3x}{(x+1)(x-1)} + \frac{2x}{x+1} - \frac{5}{x-1}\right) : \quad \rightsquigarrow \text{mcm} = (x+1)\cdot(x-1)$$

(MISMO DENOMINADOR)

$$= \frac{3x}{(x+1)(x-1)} + \frac{2x(x-1)}{(x+1)(x-1)} - \frac{5(x+1)}{(x+1)(x-1)} = \frac{3x+2x^2-2x-5x-5}{(x+1)(x-1)} =$$

$$= \frac{2x^2-4x-5}{(x+1)(x-1)} \rightsquigarrow 2x^2-4x-5=0 \; ; \quad x = \frac{4 \pm \sqrt{16+40}}{4}$$

$$\frac{1}{x^2-9} + \frac{1}{x^2-6x+9} =$$

$x=3$ $x=-3$ $\quad$ $x=3$ (DOBLE)

$$\frac{1}{(x+3)(x-3)} + \frac{1}{(x-3)^2} = \frac{(x-3)}{(x-3)^2(x+3)} + \frac{(x+3)}{(x-3)^2(x+3)}$$

$$\text{mcm} = (x-3)^2\cdot(x+3)$$

$$= \frac{x-3+x+3}{(x-3)^2(x+3)} = \frac{2x}{(x-3)^2(x+3)}$$

3) PRODUCTO Y/O COCIENTE DE FRACCIONES:

$$\frac{a}{b} : \frac{c}{d} = \frac{a\cdot c}{b\cdot d} \; ; \quad \frac{a}{b} : \frac{c}{d} = \frac{a\cdot d}{b\cdot c}$$

(PRIMERO SIMPLIFICAMOS)

$$\frac{x^4+x^3-15x^2+23x-10}{2x^2+5x-3} \cdot \frac{2x^2-7x+3}{x^3-5x^2+8x-4} = \frac{2\cdot(x-1)^2(x+5)(x-2)(x-3)(x-\frac{1}{2})}{2\cdot(x+3)(x-\frac{1}{2})(x-1)(x-2)^2} = \frac{(x-1)(x+5)(x-3)}{(x+3)(x-2)} =$$

$$\frac{(x^2+4x-5)\cdot(x-3)}{x^2+x-6} = \frac{x^3+x^2-17x+15}{x^2+x-6}$$

$x^4+x^3-15x^2+23x-10 \rightsquigarrow$

	1	1	-15	23	-10
1		1	2	-13	10
	1	2	-13	10	0
1		1	3	-10	
	1	3	-10	0	

$$(x-1)^2\cdot(x+5)(x-2)$$

$x^2+3x-10=0$ ...

$x=\frac{-3\pm7}{2}$ → $x=-5$; $x=2$

$2x^2+5x-3=0$ ... $x=-3$; $x=1/2$

$(x+3)(2x-1)$

$2x^2-7x+3=0$ ...

$(x-3)(2x-1)$

x^3-5x^2+8x-4

$(x-1)(x-2)^2$

	1	-5	8	-4
1		1	-4	4
	1	-4	4	0

$x=2$ (DOBLE)

$x^2-4x+4=0$; $x=\frac{4\pm\sqrt{16-16}}{2}=\frac{4\pm0}{2}=2$ (DOBLE)

- ECUACIONES RACIONALES.

$\frac{3}{x-4}-\frac{2}{x-3}=\frac{8}{x^2-7x+12}$; $\frac{3}{x-4}-\frac{2}{x-3}=\frac{8}{(x-4)(x-3)}$;

$x^2-7x+12=0$; $x=\frac{7\pm\sqrt{49-48}}{2}=\frac{7\pm1}{2}=$ $x=4$; $x=3$

mcm $=(x-4)(x-3)$

$\frac{3(x-3)}{}-\frac{2(x-4)}{}=\frac{8}{}$;

$$\frac{3(x-3)}{(x-4)(x-3)} - \frac{2(x-4)}{(x-4)(x-3)} = \frac{8}{(x-4)(x-3)} \quad ;$$

$3(x-3) - 2(x-4) = 8$; $3x-9-2x+8 = 8$; $3x-2x = 8-8+9$;

$x = 9$

$\frac{2}{x-2} - \frac{1}{x+2} = \frac{3}{x^2-4}$; $\frac{2}{x-2} - \frac{1}{x+2} = \frac{3}{(x+2)(x-2)}$

$$\frac{2(x+2)}{(x+2)(x-2)} - \frac{(x-2)}{(x+2)(x-2)} = \frac{3}{(x+2)(x-2)} \quad ;$$

$2(x+2) - x + 2 = 3$;

$2x+4-x+2 = 3$; $x = -3$