Ecuaciones irracionales

$3+\sqrt{3x+1}=x$

1 - DEJAMOS LA RAÍZ SOLA A UN LADO DE LA ECUACIÓN.

$\left(\sqrt{3x+1}\right)^2=(x-3)^2$

2- ELEVAMOS AL CUADRADO TODA LA ECUACIÓN

$3x+1=x^2+3^2-2\cdot x\cdot 3$;

$3x+1=x^2+9-6x$; $\quad -x^2+9x-8=0$; $\quad x=\frac{-9\pm\sqrt{81-32}}{-2}=$

$=\frac{-9\pm 7}{-2}=$ ~~$x=1$~~ ; $x=8$ ✓

3° COMPROBAMOS LAS SOLUCIONES.

$3+\sqrt{3x+1}=x$ ⇝ SI $x=1$: $\quad 3+\sqrt{3\cdot 1+1}=1$;

$5\neq 1$

↳ SI $x=8$: $\quad 3+\sqrt{3\cdot 8+1}=8$

$8=8$;

SOL: $\boxed{x=8}$

$2x-\sqrt{2x-1}=1$;

$(2x-1)^2=\left(\sqrt{2x-1}\right)^2$; $\quad (2x)^2+1^2-2\cdot 2x\cdot 1=2x-1$;

$4x^2+1-4x=2x-1$; $\quad 4x^2-6x+2=0$;

$2x^2-3x+1=0$

$(\dots) = \begin{cases} x = 2 \\ x = 1 \end{cases}$: $2 \cdot 1 - \sqrt{2 \cdot 1 - 1} = 1$

$2 \cdot 1 - 1 = 1$;

$1 = 1$

$\sqrt{2x-1} + \sqrt{x+4} = 6$;

$\left(\sqrt{2x-1}\right)^2 = \left(6 - \sqrt{x+4}\right)^2$; $2x - 1 = 6^2 + \left(\sqrt{x+4}\right)^2 - 2 \cdot 6 \cdot \sqrt{x+4}$;

$2x - 1 = 36 + x + 4 - 12\sqrt{x+4}$; $12\sqrt{x+4} = 36 + x + 4 - 2x + 1$;

$\left(12\sqrt{x+4}\right)^2 = (41 - x)^2$; $144 \cdot (x+4) = 41^2 + x^2 - 2 \cdot 41 \cdot x$;

$144x + 576 = 1681 + x^2 - 82x$; $-x^2 + 226x - 1105 = 0$;

$x = \dfrac{-226 \pm \sqrt{226^2 - 4 \cdot (-1) \cdot (-1105)}}{-2} = \dfrac{-226 \pm 216}{-2} = \begin{cases} x = 5 \\ x = 221 \end{cases}$

$\sqrt{2x-1} + \sqrt{x+4} = 6$

si $x = 5$: $(6 = 6)$ ✓

Sol : $x = 5$

si $x = 221$: $(36 \neq 6)$; ✗

Resuelve la siguiente ecuación irracional.

$\sqrt{2x^2+1} - \sqrt{x^2-3} = 2$; $\left(\sqrt{2x^2+1}\right)^2 = \left(2 + \sqrt{x^2-3}\right)^2$;

$2x^2 + 1 = 4 + \left(\sqrt{x^2-3}\right)^2 + 4 \cdot \sqrt{x^2-3}$; $2x^2 + 1 = 4 + x^2 - 3 + 4\sqrt{x^2-3}$;

$(x^2)^2 = \left(4\sqrt{x^2-3}\right)^2$; $x^4 = 16(x^2-3)$; $x^4 = 16x^2 - 48$;

$x^4 - 16x^2 + 48 = 0$; $\boxed{x^2 = t}$

$t^2 - 16t + 48 = 0$; $t = \dfrac{16 \pm \sqrt{256-192}}{2} = \dfrac{16 \pm 8}{2} = \begin{cases} t = 12 \\ t = 4 \end{cases}$

↝ $x^2 = 12$; $x = \pm\sqrt{12}$; $\boxed{x = \pm 2\sqrt{3}}$ ✓

↝ $x^2 = 4$; $\boxed{x = \pm 2}$ ✓